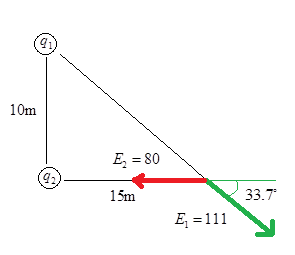
Electric Field Problems

**Problem 2 (10 pts.)**

Two charges, q1 = 4μC, q2 = -2μC, sit at the corner of a the right triangle shown below. What is the magnitude and direction of the electric field at the lower right hand vertex (specify the angle w/r to the positive x axis)?

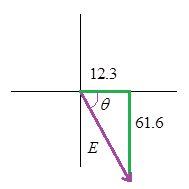
Using E = k|q|/r2 for each charge, we get the following results for E1 and E2. 33.7° angle comes from tan-1(10/15).



Then we break E1 into its components and add it to E2:



Plotting below:



from which it follows that



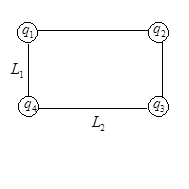
**Problem 3 (10 pts.)**

A charge q3 = -3μC is placed at the lower right hand vertex of the triangle above. What is the magnitude and direction of the force on q1­ (specify the angle w/r to the positive x axis)?

The force on the charge is simply **F** = q3**E**. Since q3 is negative, it will be in the opposite direction, i.e. 180° – 78.7° = 101.3° w/r to the positive x-axis. And the force is F = 1.88×10-4 N.

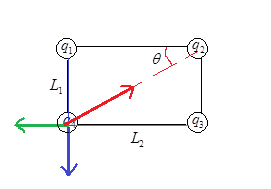
**Problem**

3 positive charges q1 = 1μC, q2 = -2μC, q3 = 3μC sit at the vertices of a rectangle with dimensions L1 = 5m and L2 = 10m. What is the electric field and force they exert on q4 = -4μC?



**Solution**

Electric fields look like this:





Adding these all together we get:



Its magnitude and direction are:



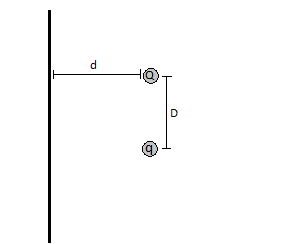
And the force on the charge q4 is:



Note the (-) sign flips the direction of the vector 180°.

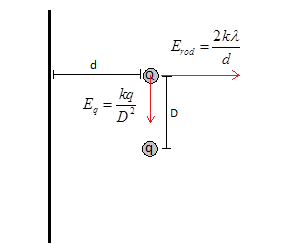
**Problem**

Suppose we have a charged infinite rod (λ = 2μC/m), and a charge q = -20μC at the positions shown, where d = 11m and D = 8m. What is the magnitude of the force on a the third charge Q = 3μC?



**Solution**

The two fields at the vicinity of Q are shown,



and so each field is given by:



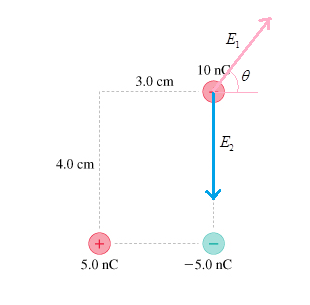
and the magnitude is:



And so the force is:



**Question 1**. (a) What is the magnitude and direction of the electric field felt by the 10nC charge? (b) What is the magnitude of the force on the 10nC charge?



The two fields are shown, and are given by:





Adding them together we get:



which has a magnitude and direction of:

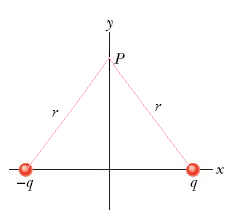


And so the magnitude of the force on the 10nC charge is:



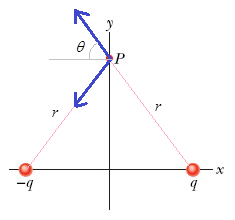
**Problem**

The figure shows two charged particles on the x axis. Both charges are a distance 5m from the origin, and the magnitude of their charge is q = 20μC. What is the magnitude and direction of the field at point P, which is a distance 2m away from the origin?



**Solution**

First we’ll draw in the field vectors.



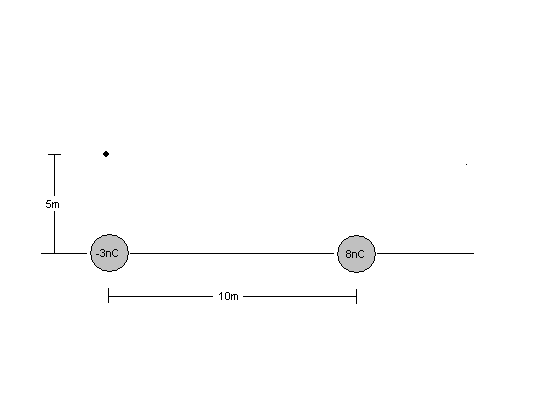
The net field is:



angle is 21.8 degrees.

**Problem**

Suppose have that charge and an 8nC charge 10m down the x axis. What is field 5m above (-3nC) charge?

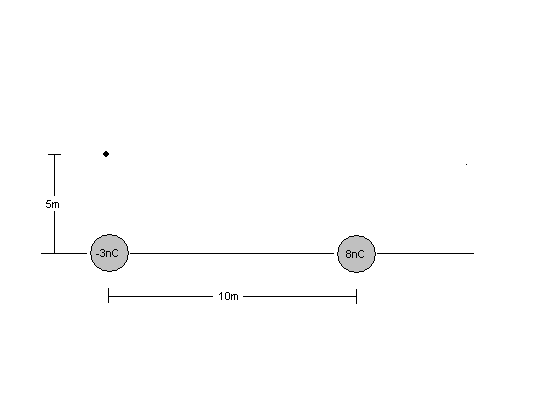


**Solution**

First we calculate the field due to the -3nC charge, which we’ll call q1. From before we have,



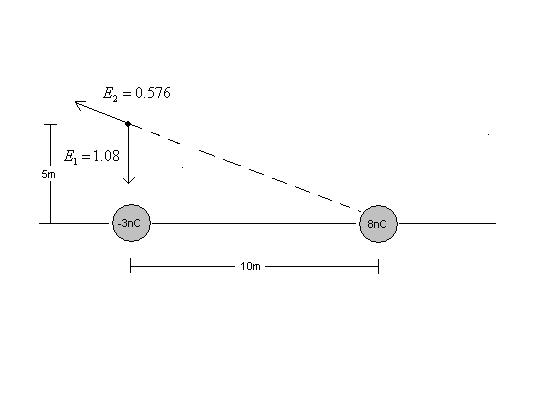
Drawing this in we get,



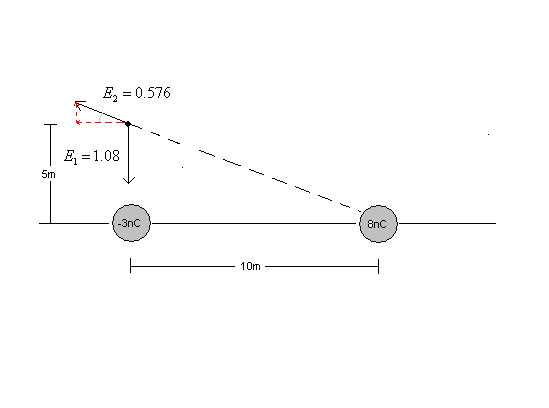
while, (ask for direction) using the Pythagorean theorem, **E**2 is:



and so we have,



Now add the two **E** vectors together. At this point it is probably best to convert **E**2 to the unit basis. This can be done by drawing the vector out (already done I imagine), and finding its components along the axes. Then the addition can proceed as usual by vector addition. Can put in polar form if want. So I would break **E**­2 into its components,



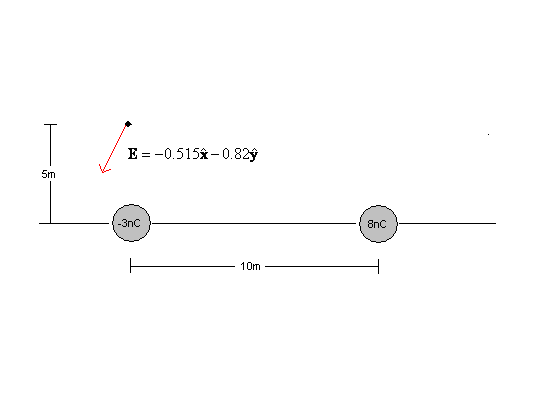
The grey-angle is θ = tan-1(5/10) = 26.6 degrees. Therefore:



And therefore,

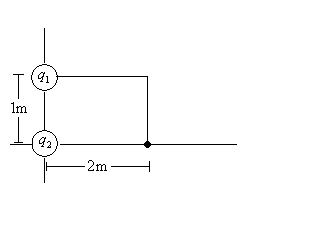


At this point can draw in the vector, or can calculate polar form.



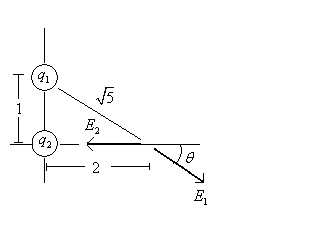
**Problem**

Suppose that , . Determine the magnitude and direction of the electric field (in N/C) at the lower right hand corner of the rectangle below.



**Solution**

So both q1 and q2 set up electric fields at the lower left hand corner of the rectangle. These are shown in bold.



Before we go on to calculate them, we’ll first note that the hypotenuse of the triangle drawn is



Now note that the angle E1 makes with respect to the horizontal is:



Now let’s consider the magnitude of the fields…we will use the formula below.

, so





And now we form the vectors:



And now add the vectors to get the net field.



Since both components are negative, the vector points in the lower left hand corner (3rd quadrant). The magnitude of the field is:

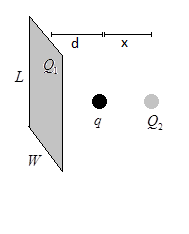


Direction is:



**Problem 4 (10 pts.)**

A rectangular sheet with dimensions W = 3m, L = 4m, and total charge Q1 = 5μC sits a distance d = 1m away from some charge q. Desiring the net force on q to be zero, we place another charge Q2 = 2μC a distance x away from q. What is *x*?

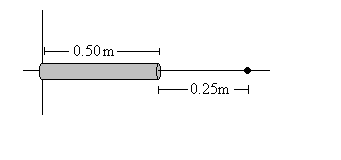


In order for the net force to be zero, the net **E** field must be zero. This field is given by:



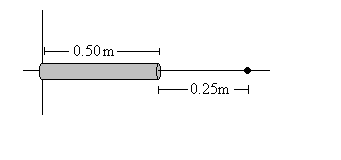
**Problem**

Calculuate the electric field at the point indicated, supposing that a total charge of 2μC is uniformly distributed over the length of the wire. Be sure to show the work, from the integral expression for **E** to the solution.



**Solution**

Calculuate the electric field at the point indicated, supposing that a total charge of 2μC is uniformly distributed over the length of the wire. Be sure to show the work, from the integral expression for E to the solution.



Let L = 0.50m, and r = 0.75m. Then,



Now λ = 2μC/0.5m = 4×10-6C/m. And so:



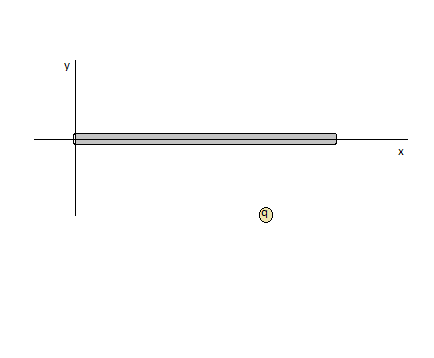
**Problem 6 (10 pts.)**

A 0.75μm diameter droplet of oil, having a charge of +2e, is suspended in midair between two horizontal plates of a parallel plate capacitor. The upward electric force on the droplet is exactly balanced by the downward force of gravity. The oil has a density of 860 kg/m3, and the capacitor plates are 7.5mm apart. What must be the potential difference between the plates to hold the droplet in equilibrium?

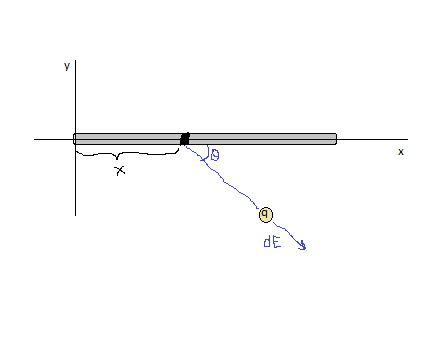
We have:



**Question 1**. A q = -20nC charge zips past a wire segment of length ℓ = 10cm with linear charge density 30μC/m. If the charge is presently at coordinate (x0,y0) = (7.5cm,-5cm) what is the magnitude and direction (CCW w/r to + x axis) of the force on the charge? Be sure to show all work – formulas squirreled away in your calculator don’t get credit ☺.



First have to find E field at that location. So isolate piece of charge at coordinate x, width dx. Then the field d**E** it creates is:





and so

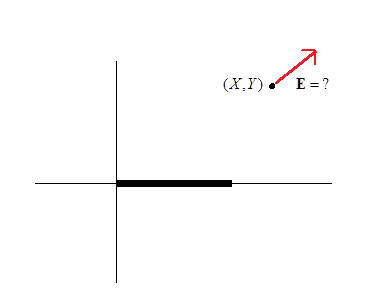


And so the force on that charge would be:



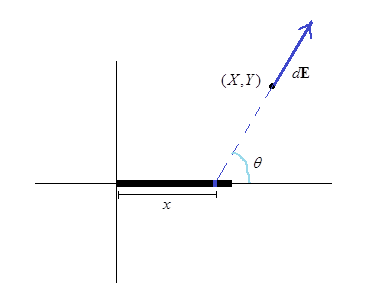
**Problem**

Consider a charged rod of length L and charge Q evenly distributed along its length, pictured as shown. What is the electric field vector at arbitrary coordinate (X,Y) in the x-y plane?



**Solution**

We start by isolating a tiny point charge dq = λdx at coordinate x on the rod, and calculating the field created by that point charge at coordinate (X,Y)



For d**E** we have:



and now we integrate over x from 0 to L



To do the second anti-derivative … we can look it up or do trig substitution. Considering 1 + tan2θ = sec2θ, Let X – x = Ytanθ, and then d(X – x) = -dx = Ysec2θdθ. And then we have:



And now from fact that X – x = Y tanθ, we have tanθ = (X-x)/Y → sinθ = (X-x)/√[(X-x)2 + Y2]. So now we have:

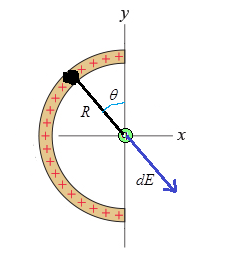


and finally, filling in λ = Q/L



**Question 2**. Consider a semicircular ring with radius R = 5cm and charge Q = 10μC evenly distributed throughout. What is the force on a charge q = 20μC at the center of the semicircular ring.

First we must find the field created by the half-ring. And then we can get the force. To that end we label a piece of charge at coordinate θ and width dθ (and so length ds = Rdθ). The charge of that piece will be dq = λds = (Q/πR)(Rdθ) = Qdθ/π. And so the magnitude of the field created at the center of the ring is dE = kdq/r2 = kQdθ/πR2.



and the field d**E** will be given by:



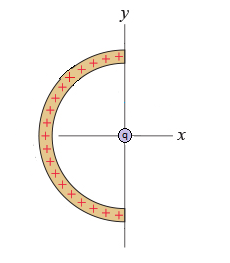
Integrating to get the total field:



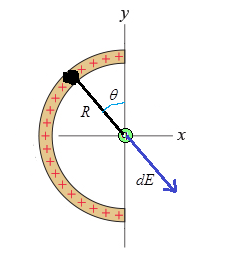
Then the force is given by:



**Question 1**. Consider a semicircular ring with radius R = 3cm and charge Q = 15μC evenly distributed throughout. What is the force on a charge q = -5μC at the center of the semicircular ring.



First we must find the field created by the half-ring. And then we can get the force. To that end we label a piece of charge at coordinate θ and width dθ (and so length ds = Rdθ). The charge of that piece will be dq = λds = (Q/πR)(Rdθ) = Qdθ/π. And so the magnitude of the field created at the center of the ring is dE = kdq/r2 = kQdθ/πR2.



and the field d**E** will be given by:



Integrating to get the total field:



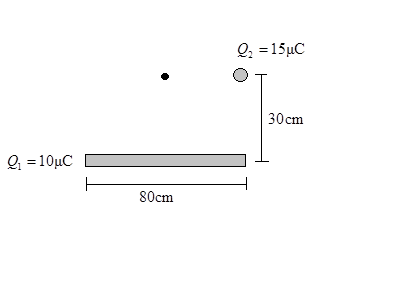


Then the force is given by:



**Question 1 (20 points)**

An 80cm rod is uniformly charged to Q1 = 10μC, and a point charge Q2 = 15μC sits 30cm above its right end point. What is the magnitude and direction of the electric field at the point 30cm above the midpoint of the rod? Specify the angle with respect to the x-axis.



First have to find the net electric field. That due to rod is:



Field due to point charge Q2 is:



The net field is:

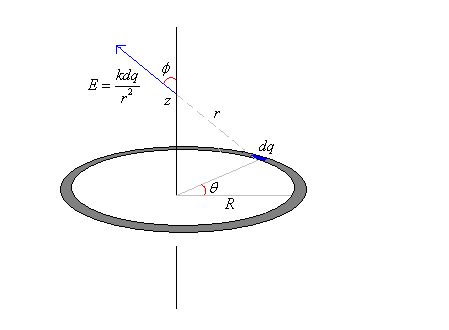


**Problem**

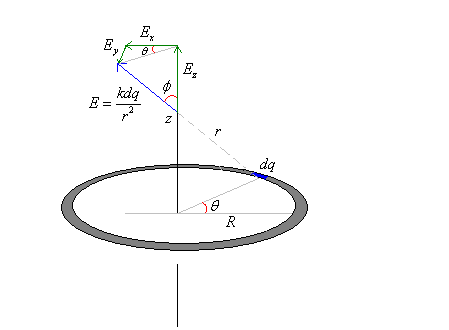
Consider a ring of charge, radius R, and charge density λ = Q/2πR. Let’s calculate the field a point z above its center.

**Solution**

So first we isolate a piece of charge dq, and draw the field E, which it creates.

.

Now we want to break it into its components along the x, y, and z axes.



Now let’s write E in vector form.



And then we sum/integrate over all charges,



Now clearly the variable which is changing as we go around the circle is θ, so we need to express dq in terms of θ. dq = λ(Rdθ) where Rdθ is the arc length of the piece of circle in which dq resides. Note that r and φ don’t depend on θ however. So we have,



the first two integrals are 0 and the last is just 2π, and so we have,



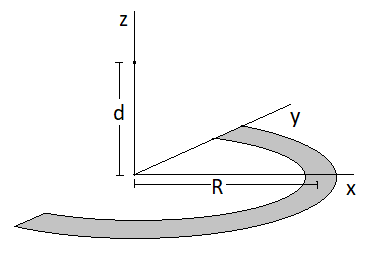
We should see the cancellation of the Ex and Ey components anyway. They must be 0 due to symmetry.

Now , and  so we have:



**Problem**

Consider an infinite long straight wire with linear charge density λ. Let’s calculate the field at a distance R away. Let Q = -30pC, R = 10cm, d = 15cm.

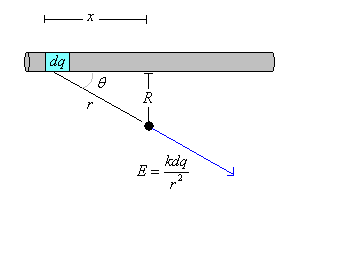


Field is:

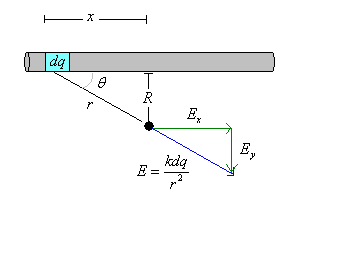


**Solution**

So first draw the wire. Isolate an arbitrary dq, and draw the field coming from that point at the place where we’re trying to calculate the field.



Now break E into its components.





and now sum over all the charges,



Now as we go from one end of the wire to the other, θ and r both change. Let’s put both in terms of x.



additionally, dq = λdx. So filling these in, and summing from x = -∞ to x = ∞, we have,



the first integral is odd w/r to x and so it vanishes. We should see this anyway; the net E will have no x-component because of symmetry. So continuing, we have to evaluate the integral. We could just look it up in an integral table, or we could compute the anti-derivative by using a trig substitution. But since you’re in principle familiar with those things, I’m going to do something a little bit different, which is quite frequently useful when working with integrals, especially when an anti-derivative cannot be found, or is time consuming to calculate. We’re going to factor out the R from the integral. Make the substitution x = Ry, then dx = Rdy.



The y-integral is just a number with no R-dependence and we can use a calculator to evaluate it. The answer we get is 2. So



and putting this in our expression we have,



and so our final result is:



**Problem**

Now let’s calculate the field along central axis of a charged disk.

**Solution**

Borrowing from ring example, we have:

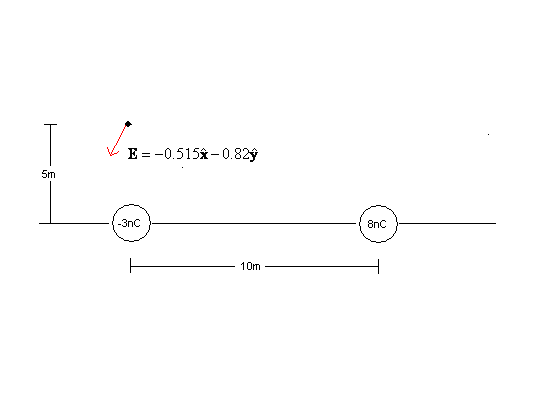


Note that if disk is infinitely wide, then we get:



**Problem**

Consider the electric field, **E**, below at the indicated point. If an electron is placed there, what force would it experience – give magnitude and direction, θ, from the + x-axis.



**Solution**

In the figure directly above, if an electron is placed at the point indicated, where **E** is given, what will be the magnitude of the force that it experiences ( \_×10-19N)?

**F = E**|q| opposite the direction of **E** since q is negative.

Now,



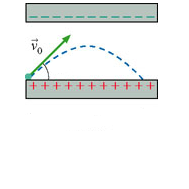
and the direction of **E** is:



And so the force is:



**Question 2**. In the picture below, the top plate has total charge Q = -4nC, and area A = 25cm2. The bottom plate has opposite charge but equal area. If an electron is fired with velocity v0 = 4×107 m/s, at an angle of 30°, how far will it go to the right before it hits the bottom plate again?



First we gotta find the field, you know? So:



Then the acceleration of our electron is:



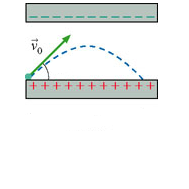
The time it takes to hit the plate will be given by:



and then we can determine the horizontal distance traveled…



**Question 3**. In the picture below, the top plate has total charge Q = -8nC, and area A = 25cm2. The bottom plate has opposite charge but equal area. If an electron is fired with velocity v0 = 5×107 m/s, at an angle of 60°, how far will it go to the right before it hits the bottom plate again?



First we gotta find the field, you know? So:



Then the acceleration of our electron is:



The time it takes to hit the plate will be given by:

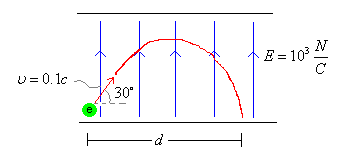


and then we can determine the horizontal distance traveled…



**Problem**

Consider the following problem. Suppose we inject an electron into a constant electric field at a speed of v = 0.1c at an angle of 30 degrees w/r to the horizontal, maintained by two oppositely charged plates (the top plate would be negative, and the bottom positive). How far will it go before it hits the bottom plate again?



**Solution**

We label the distance, *d*, and we use the kinematics equations from physics 1, namely,



Now let Δx be our displacement d, then t is the time it takes to make that displacement, Δy will be 0 since it begins/ends at same height. c is the speed of light (3×108m/s). So the x-component of the velocity is:



and the y-component of the velocity is:



What is the acceleration? There is no acceleration in the x-direction since there is no force in that direction. What is the acceleration in the y direction? Well, the force on the electron is downward. We know this because electrons are negatively charged and will therefore experience a force in the direction opposite to **E**. We can directly compute the acceleration using N2L,



The force is supplied by the electric field so,



Therefore,



which again confirms that the acceleration is in the y-direction, and downward. So filling in all of these values, we have,



We can solve the last equation for t,



and then plug into the top equation for d.



11. If a 1.5kg, 2C charge is traveling with velocity 10m/s to the right in a field **E** = 7N/C to the left, how far will it go before it comes to rest?

Use conservation of energy,



**Problem**

Consider the centripetal motion of the e- around a Hydrogen nucleus. Calculate its velocity assuming a certain radius r = 1A.

**Solution**

Now for any orbit, r and υ are related via N2L,

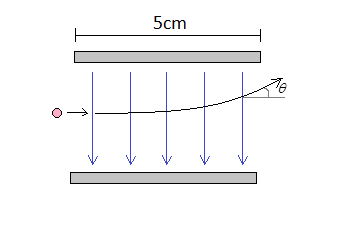


which yields,



**Question 3 (20 points)**

A particle with charge q = -23nC, and mass m = 7×10-20kg, enters the region between two equal and oppositely charged (Q = ± 3fC) square (5cm × 5cm) plates with a horizontal velocity v = 67km/s. At what angle θ will the particle deflect upon exiting the region? You may use the infinite plate approximation for the fields of the square plates.



So E is given by



And so the acceleration of the particle will be:



It will take a time t to exit the region, given by:



So its y velocity component will be:



and so the deflection angle will be:

